NOTATION

v, flow velocity of the fluid; U, instantaneous temperature; x, y, z, moving coordinates; a, thermal conductivity of the fluid; Γ, contour of the channel; U_0, initial temperature of the fluid; U_Γ, temperature of the channel wall; Z, dimensionless length of the channel; Θ, dimensionless temperature; G, region; $\bar{G}$, region with boundary Γ; φ, fluidity of the fluid; τ, shear stress; τ_x and τ_y, shear stress components; ΔP, pressure differential per unit length of channel; ψ, a function that is a solution of the Dirichlet problem in the Poisson equation; $\chi = (\chi_1, \chi_2)$, a point of two-dimensional Euclidean space; h_β, step of the grid ω_h; γ_h, set of boundary nodes; C_β, a line passing through the interior nodes; ω_h, set of all regular nodes; $L_\beta U$, Laplacian operator; Λ_β, difference operator; h_β^*, distance from the nonregular node χ to the boundary node $\chi^{(+1\beta)}$ or $\chi^{(-1\beta)}$; τ^*, step of the grid along the z coordinate; $h_{i_\beta}^*$, distance from the near-boundary nodes $\omega_{h,\beta}^*$ to the boundary nodes $\gamma_{h,\beta}$; N_{i_β}', number of the left boundary nodes in the matrix in the direction of χ_β; N_{i_β}'', number of the right boundary nodes in the matrix in the direction of χ_β; φ_0, fluidity of the fluid for $\tau \to 0$; K and m, rheological constants.

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UNSTEADY HEAT TRANSFER IN A MICROPOLAR FLUID FLOWING IN A PLANE CHANNEL

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Heat transfer in a micropolar fluid flowing in a plane channel following an abrupt change in the wall temperature is investigated. The obtained results indicate that in several cases the fluid microstructure has a considerable effect on the main heat-transfer characteristics.

The theory of heat-conducting micropolar fluids (MPF) [1] can be used to characterize the hydrodynamic and thermal processes in several microstructural fluids (liquid crystals, suspensions, blood, etc.) with due consideration of the spinning of the particles in the medium. The hydrodynamics of MPF has now been widely investigated. There have been investigations of free convection, and also of steady heat transfer involving forced convection, where it was discovered that the microstructure of the fluid affects the characteristics of heat transfer in it. So far, however, due attention has not been paid to such an important practical problem as unsteady heat transfer in MPF.

We consider the following problem. A heat-conducting MPF flows between plane parallel plates separated by a distance 2h. Let the temperature of plates and MPF over the whole length of the channel be constant and equal to T_0. At a certain instant the temperature of the plates is abruptly altered and becomes equal to $T_j \neq T_0$. We determine the temperature field over the cross section and length of the channel in relation to time. We neglect energy dissipation, the compressibility of the MPF, axial heat conduction, and mass forces and moments. We regard the hydrodynamic velocity profile as stabilized, and the physical properties of the MPF as constant. The coordinate origin is on the central line at the entrance section of the channel, which has temperature T_0. The central line coincides with the

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x axis, and the normal to the planes coincides with the y axis. The components of the velocity vector $\mathbf{v}$ and the microrotation vector $\boldsymbol{\nu}$ have the form

$$v_k = \{v_x(y),\ 0,\ 0\},\ \nu_k = \{0,\ 0,\ \nu_z(y)\}.$$

The energy equation and boundary conditions in this case can be written as

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial y^2} - v_x\frac{\partial T}{\partial x}, \tag{1}$$

$$T(x,\ y,\ 0) = T_0,\ T(0,\ y,\ t) = T_0, \tag{2}$$

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0,\ T(x,\ \pm h,\ t) = T_j,$$

where a is the thermal diffusivity.

The solution of the system of differential equations representing the hydrodynamics in the given approximation [1]:

$$(\mu+\varkappa)\frac{d^2 v_x}{dy^2} + \varkappa\frac{d\nu_z}{dy} = \frac{dP}{dx}, \tag{3}$$

$$\gamma\frac{d^2\nu_z}{dy^2} - \varkappa\left(\frac{dv_x}{dy} + 2\nu_z\right) = 0,$$

with boundary conditions

$$\mathbf{v}(\pm h) = 0,\ \boldsymbol{\nu}(\pm h) = \frac{\alpha}{2}(\mathrm{curl}\,\mathbf{v})|_{y=\pm k} \tag{4}$$

leads to the following velocity profile:

$$v_x = v_0\left[1 - \tilde{y}^2 + \delta\left(\frac{\mathrm{ch}\,k\tilde{y}}{\mathrm{ch}\,k} - 1\right)\right], \tag{5}$$

where

$$v_0 = -\frac{dP}{dx}\frac{h^2}{2\mu+\varkappa},\quad \delta = \frac{2\varkappa(1-\alpha)}{2\mu+\varkappa(2-\alpha)}\frac{\mathrm{cth}\,k}{k},$$

$$k^2 = \frac{2\mu+\varkappa}{\mu+\varkappa}\frac{\varkappa}{\gamma}h^2,\quad \tilde{y} = \frac{y}{h}.$$

In (3) and (4) α is a parameter characterizing the interaction of the fluid particles with the boundaries and with one another ($0 \leqslant \alpha \leqslant 1$). We substitute (5) in (1) and bring the obtained equation and boundary conditions (2) to dimensionless form:

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}} = \frac{\partial^2\Theta}{\partial\tilde{y}^2} - f(\tilde{y})\frac{\partial\Theta}{\partial\tilde{x}}, \tag{6}$$

$$\Theta(\tilde{x},\ \tilde{y},\ 0) = 0,\ \Theta(0,\ \tilde{y},\ \mathrm{Fo}) = 0,$$

$$\left(\frac{\partial\Theta}{\partial\tilde{y}}\right)_{\tilde{y}=0} = 0,\ \Theta(\tilde{x},\ \pm 1,\ \mathrm{Fo}) = 1, \tag{7}$$

where

$$\Theta = \frac{T-T_0}{T_j-T_0},\quad \tilde{x} = \frac{4}{3\mathrm{Pe}}\frac{x}{h},\quad \mathrm{Pe} = \frac{v_{av}^{(N)}2h}{a},\quad \mathrm{Fo} = \frac{at}{h^2},$$

$f(\tilde{y}) = 1 - \tilde{y}^2 + \delta\left(\frac{\mathrm{ch}\,k\tilde{y}}{\mathrm{ch}\,k} - 1\right)$, $v_{av}^{(N)}$ is the average velocity of a Newtonian fluid with shear viscosity $\mu + \varkappa/2$ in a channel of height 2h.

We also investigate the effect of the fluid microstructure on such an important heat-transfer characteristic as the local Nusselt number, which by definition relates the local heat-transfer coefficient to the temperature difference $T - \bar{T}$. Then

$$\mathrm{Nu} = -\frac{2}{\bar{\Theta}}\left(\frac{\partial\Theta}{\partial\tilde{y}}\right)_{\tilde{y}=1},$$

where

$$\bar{\Theta} = \left[\frac{2}{3} + \delta\left(\frac{\operatorname{cth} k}{k} - 1\right)\right]^{-1} \int_0^1 \Theta f(\tilde{y})\, d\tilde{y}.$$

We solve the problem (6), (7) numerically by a finite-difference method in the region of the variables $G = \{0 \leqslant \tilde{x} \leqslant 1,\ 0 \leqslant \tilde{y} \leqslant 1,\ 0 \leqslant \mathrm{Fo} \leqslant 1\}$. In the following expression of the algorithm of the solution we omit for simplicity the tilde above the dimensionless variables.

In region G we construct a uniform net with steps h_1, h_2, τ

$$\bar{\omega} = \{x_i = ih_1,\ y_j = jh_2,\ \mathrm{Fo}_n = n\tau,\ i = 0,\ 1,\ \ldots,\ N_1,$$
$$j = 0,\ 1,\ \ldots,\ N_2,\ n = 0,\ 1,\ \ldots,\ N,\ h_1N_1 = 1,\ h_2N_2 = 1,\ \tau N = 1\}.$$

We denote the value of the function $\Theta(x, y, \mathrm{Fo})$ at the point $(x_i, y_j, \mathrm{Fo}_n)$ by $\Theta_{i,j,n}$. On the net $\bar{\omega}$ we approximate the initial problem by the following implicit difference scheme:

$$\frac{\Theta_{i,j,n} - \Theta_{i,j,n-1}}{\tau} = \frac{\Theta_{i,j+1,n} - 2\Theta_{i,j,n} + \Theta_{i,j-1,n}}{h_2^2} - f(y_j)\frac{\Theta_{i,j,n} - \Theta_{i-1,j,n}}{h_1},$$
$$i = \overline{1,\ N_1 - 1},\ j = \overline{1,\ N_2 - 1},\ n = \overline{1, N},$$
$$\Theta_{i,j,0} = 0,\ \Theta_{0,j,n} = 0,\ \Theta_{i,N_1,n} = 1,\ \Theta_{i,1,n} = \Theta_{i,0,n}.$$

To realize the proposed difference scheme we use an iteration process of the form

$$\left.\begin{aligned} &\frac{\Theta_{i,j,n}^{(s+0,5)} - \Theta_{i,j,n-1}}{\tau} = \frac{\Theta_{i,j+1,n}^{(s+0,5)} - 2\Theta_{i,j,n}^{(s+0,5)} + \Theta_{i,j-1,n}^{(s+0,5)}}{h_2^2} - \\ &- f(y_j)\frac{\Theta_{i,j,n}^{(s)} - \Theta_{i-1,j,n}^{s}}{h_1},\ j = 1,\ 2,\ \ldots,\ N_2 - 1, \\ &\Theta_{i,1,n}^{(s+0,5)} = \Theta_{i,0,n}^{(s+0,5)},\ \Theta_{i,N_1,n}^{(s+0,5)} = 1. \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} &\frac{\Theta_{i,j,n}^{(s+1)} - \Theta_{i,j,n-1}}{\tau} = \frac{\Theta_{i,j+1,n}^{(s+0,5)} - 2\Theta_{i,j,n}^{(s+0,5)} + \Theta_{i,j-1,n}^{(s+0,5)}}{h_2^2} - \\ &- f(y_j)\frac{\Theta_{i,j,n}^{(s)} - \Theta_{i-1,j,n}^{s}}{h_1},\ i = 1,\ 2,\ \ldots,\ N_1, \\ &\Theta_{0,j,n}^{(s+1)} = 0. \end{aligned}\right\} \tag{9}$$

Here s is the number of the iteration, (s + 0.5) is the number of the intermediate iteration (subiteration).

The system of equations (8) is solved by the pivotal method with fixed values of $i = \overline{1, N_1}$. The solution is then made more accurate and is found in explicit form from the system of equations (9) with $j = 1,\ N_2 - 1$. The calculation procedure on each n-layer is repeated until the inequality

$$\max_{i,j} |\Theta_{i,j,n}^{(s+1)} - \Theta_{i,j,n}^{(s)}| \leqslant \varepsilon_{\mathrm{T}}$$

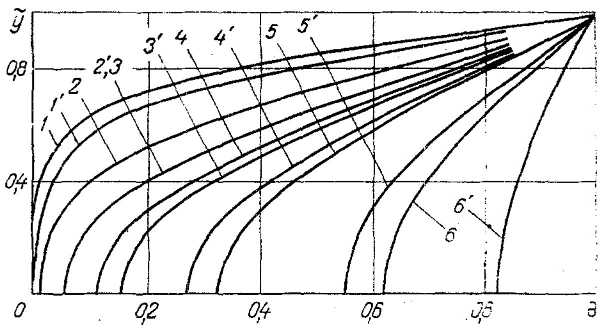

Fig. 1. Temperature field Θ along channel with Fo = 1.0; $\tilde{x}$ = 0.01 (1, 1'); 0.04 (2, 2'); 0.07 (3, 3'); 0.1 (4, 4'); 0.2 (5, 5'); 0.4 (6, 6'): 1-6) ε = 0; 1'-6') k = 0.1; ε = 10/7.

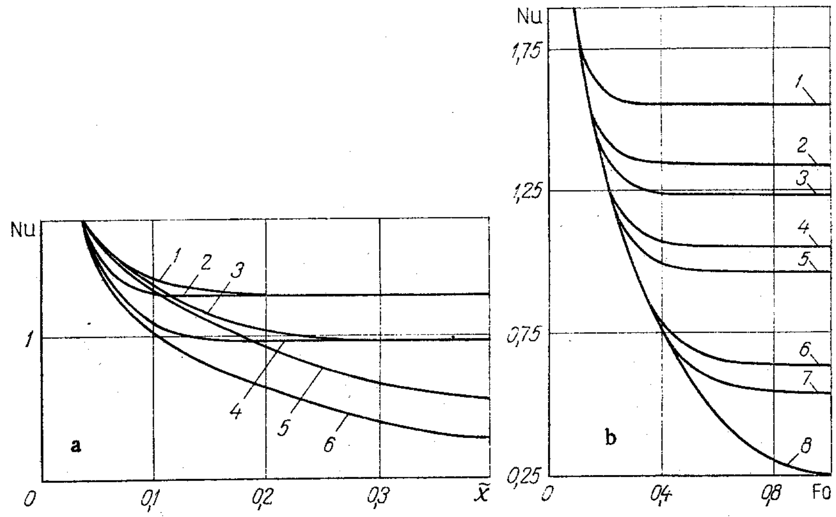

Fig. 2. Time dependence of Nu along channel: a) Fo = 0.2 (1, 2); 0.3 (3, 4); 1.0 (5, 6): 1, 3, 5) $\varepsilon = 0$; 2, 4, 6) $k = 0.1$; $\varepsilon = 10/7$; b) $\tilde{x} = 0.07$ (1, 3); 0.1 (2, 4); 0.2 (5, 6); 0.4 (7, 8): 1, 2, 5, 7) $\varepsilon = 0$; 3, 4, 6, 8) $k = 0.1$; $\varepsilon = 10/7$.

is satisfied, where ε_T is the prescribed accuracy of convergence of the iteration process.

The problem was solved on a BÉSM-6 computer for different values of k and ε. Consideration of the microstructure led to maximum variation of the theoretical values of v_x and Θ when $\alpha = 0$. A change in ε with k constant is formally equivalent in effect on the results to a particular change in α with k and ε constant and, hence, in subsequent calculations α is assumed equal to zero throughout.

The results of the numerical calculation are given in the figures. There is a considerable difference in the temperatures and Nusselt numbers calculated with the aid of Newtonian fluid theory with microstructure neglected ($T^{(N)}$ and $Nu^{(N)}$) and within the framework of MPF theory ($T^{(MP)}$ and $Nu^{(MP)}$). This is particularly clearly revealed by Figs. 1 and 2. With increase in channel length $\tilde{x}$ consideration of the microstructure leads to an increase in the indicated difference. At certain values of $\tilde{x}$, depending on the values of Fo, Nu reaches a maximum and then decreases. At low k and certain values of $\varepsilon = \varkappa/(\mu + \varkappa/2)$, $\tilde{x}$ and Fo with due consideration of the microstructure Nu and the temperature on the channel axis are reduced by a factor of two or more. The difference of $T^{(N)}$ and $Nu^{(N)}$ from $T^{(MP)}$ and $Nu^{(MP)}$, calculated for the same points in the channel and the same time, is greater, the smaller k, i.e., the smaller the transverse dimension of the channel (Fig. 3). We note that the change in $T^{(MP)}$ and $Nu^{(MP)}$ with reduction of k occurs mainly in the region $0.5 < k < \infty$. Further reduction of k leads to a small relative change in these quantities (Figs. 3, 4).

We introduce the length of the thermal initial portion l_{it}, defined as the distance from the entrance section at which for a specific value of the time Fo the Nusselt number assumes a constant value with prescribed accuracy. If the accuracy is equal to 5%, for instance, then for $k = 0.1$ and $\varepsilon = 10/7$ with Fo = 0.3, $l_{it}^{(N)}/l_{it}^{(MP)} \simeq 1.8$ (Fig. 2a).

We define the time of onset of the steady-state Fo_{st} as the time on the elapse of which Nu differs by not more than 5% from its value when $Fo \to \infty$. It is apparent then from Fig. 2b that in the region $\tilde{x} = 0.07$–0.2 consideration of the microstructure at $k = 0.1$ and $\varepsilon = 10/7$ leads to an increase in Fo_{st} by 30-60% (Fig. 2b). The situation is the same when the temperature field is stabilized at a specific distance from the entrance. For instance, when Fo = 0.6 the difference between the temperature on the axis at a distance from the entrance $\tilde{x} = 0.4$, calculated within the framework of Newtonian fluid theory, and the steady value is 1.5% (in Fig. 4 the curves corresponding to Fo = 1.0 coincide with the curves for $Fo \to \infty$). When $k = 0.1$ and $\varepsilon = 10/7$, however, for the same $\tilde{x} = 0.4$ and Fo = 0.6 the temperature profile is still far from the steady profile. If $\Theta = \Theta(\tilde{x}, \tilde{y}, \mathrm{Fo})$, then

$$\frac{\Theta^{(N)}(0.4;\ 0;\ \infty)}{\Theta^{(N)}(0.4;\ 0;\ 0.6)} = \frac{\Theta^{(MP)}(0.4;\ 0;\ \infty)}{\Theta^{(MP)}(0.4;\ 0;\ 0.87)}.$$

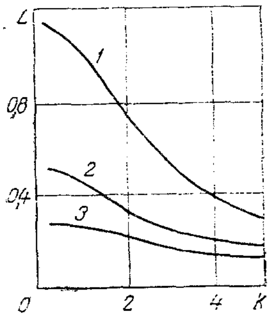

Fig. 3

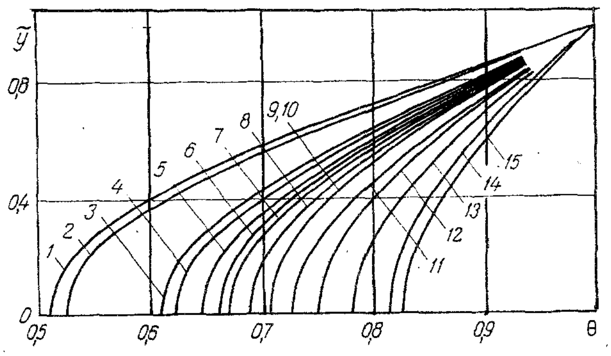

Fig. 4

Fig. 3. Dependence of $L = Nu^{(N)}/Nu^{(MP)} - 1$ on k for Fo = 1.0 and $\varepsilon = 10/7$; $\tilde{x} = 0.4$ (1); 0.2 (2); 0.1 (3).

Fig. 4. Dependence of temperature field Θ on time Fo and parameters k and ε for $\tilde{x} = 0.4$: 1) k, any value; ε = 0, Fo = 0.4; 2) 0.1; 10/7; 0.4; 3) any value; 0; 0.6; 4) any value; 0; 1; 5) 3; 0.4; 0.6; 6) 1; 0.4; 0.6; 7) 5, 10/7; 0.6; 8) 3; 10/7; 0.6; 9) 0.1; 10/7; 0.6; 10) 1; 10/7; 0.6; 11) 4; 10/7; 1; 12) 3; 10/7; 1; 13) 2; 10/7; 1; 14) 1; 10/7; 1; 15) 0; 10/7; 1.

Thus, in the given case the time for establishment of the steady value of the temperature on the axis, accurate to within 1.5%, is increased by 45%.

Ariman et al. [2] cite numerical values of the coefficient $\varkappa$, μ, and γ for blood with a red corpuscle content of 40%. We note here that the boundary condition of constancy of microrotation close to the wall (absence of moment stresses) corresponds well with the experimental results. A comparison of the velocity profile (5), obtained with boundary conditions of general form (4), with that obtained in [3] shows that when

$$\alpha = \frac{2(\mu+\varkappa)(k - \operatorname{th} k)}{2(\mu+\varkappa)k - \varkappa \operatorname{th} k} \tag{10}$$

the second boundary condition from (4) implies the absence of moment stresses on the wall. The use of (10) and numerical values of the viscosity coefficients for blood shows that when Fo = 0.3, $l_{it}^{(N)} > l_{it}^{(MP)}$ by 17%, and when $\tilde{x} = 0.1$, $Fo_{st}^{(N)} < Fo_{st}^{(MP)}$ by 16%.

The above analysis of the results of a numerical calculation indicates that calculation of many characteristics of unsteady heat transfer for the flow of microstructural fluids in capillary channels within the framework of Newtonian fluid theory can lead to incorrect results. The use of MPF theory for the calculation enables us to take into account the effect of microstructure on the hydrodynamics and heat transfer.

NOTATION

T, temperature; v, velocity; ν, microrotation; t, time; a, thermal diffusivity; $\varkappa$, μ, γ, coefficients of viscosity of micropolar fluid; dP/dx, pressure gradient; 2h, distance between planes forming channel; Fo and Nu, Fourier and Nusselt numbers.

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